

Real Analysis Qualifying Exam

Answer all four questions. In your proofs, you may use any major theorem, except the result you are trying to prove (or a variant of it). State clearly what theorems you use. All four questions are worth the same number of points. Good luck.

Notation: In the questions below,

Question 4. Denote by \mathcal{A} the smallest algebra of subsets of \mathbb{R} that contains all bounded intervals. Denote by \mathcal{A}^* the collection of countable unions of sets in \mathcal{A} . Denote by μ^* the outer measure on the power set $\mathcal{P}(\mathbb{R})$ induced by the premeasure on \mathcal{A} that assigns to any bounded interval its Euclidean length, and to any unbounded interval 1 .

- Let $E \subseteq \mathbb{R}$. What does " E is μ^* -measurable" (i.e. outer measurable) mean?
- How is the collection of μ^* -measurable sets related to the collection of \mathcal{A}^* -measurable sets?
- Prove that for any $E \subseteq \mathbb{R}$ and any $\epsilon > 0$, there exists $A \in \mathcal{A}^*$ with $E \subseteq A$ and $\mu^*(A) - \mu^*(E) < \epsilon$.