ALGEBRA QUALIFYING EXAM SYLLABUS 2015

Recommended Textbooks

Abstract Algebra by Dummit and Foote
Introduction to Commutative Algebra by Atiyah and Macdonald
Algebra by Lang.

Topics covered

I. Groups.

Group actions, orbit-stabilizer theorem, Sylow theorems, semi-direct products, Jordan-Holder theorem.
Examples: symmetric, alternating, dihedral groups, general and special linear groups.

II. Linear algebra.

Modules over a PID, elementary divisor theorem.
Invariant factors and similarity classes of matrices.
Jordan and rational canonical forms, Cayley-Hamilton theorem.

III. Fields.

Polynomial rings, Gauss lemma, Eisenstein criterion.
Finite elds: construction, classi cation, structure of the units.
Normal and separable extensions, Galois groups and the Galois correspondence.
Computing Galois groups of low degree extensions, cyclotomic elds.
Discriminants, symmetric polynomials, insolvability of the general quintic.
Transcendence degree.

IV. Rings and commutative algebra.

Noetherian and Artinian rings and modules.
Discrete valuation rings, local rings, localization, Nakayama's lemma.
Primary decomposition.
Integral extensions. Going-up and going-down theorems.

V. Modules and homological algebra.

Tensor product of modules and algebras.
Exact sequences. Projective, injective, at modules.
Complexes. Projective and injective resolutions, Ext and Tor.
Localization of modules.

VI. Algebraic Geometry.

Zariski topology, Spec of a commutative ring, algebraic sets in a ne space.
Hilbert's Nullstellensatz, Noether Normalization, Krull dimension.

VII. Algebraic Number Theory.

Algebraic integers, discriminants.
Prime factorization in Dedekind rings.