

Algebra Qualifying Exam
 Fall 2015
 3 hours

1. Classify groups of order 55 up to isomorphism. Give a presentation for each of the groups in your classification.
2. Let $R = \mathbb{C}[X; Y]$ and consider the ideal $I = (X; Y)$ as an R -module.

(a) Construct an exact sequence of R -modules

$$0 \rightarrow R \rightarrow R \rightarrow I \rightarrow 0$$

(b) Prove that the sequence you constructed is not split.

3. Consider the ideal

$$I = (X^2 - Y; Y^2 - X) \subseteq \mathbb{C}[X; Y]$$

Find all maximal ideals of the quotient $\mathbb{C}[X; Y]/I$. (Find means give a set of generators.)

4. How many Sylow p -subgroups are there in $GL_2(\mathbb{F}_p)$?
5. Suppose K is an extension of \mathbb{Q} of degree n , and let $\sigma_1, \dots, \sigma_n : K \rightarrow \mathbb{C}$ be the distinct embeddings of K into \mathbb{C} . Let $\sigma \in \text{Gal}(K/\mathbb{Q})$.