§1: Algebraic Topology

(1) Let $X = T^2 \# X$ be ... corresponding to the subgroup $H = \langle a, b^2, bab^{-1} \rangle \subset \pi_1(X)$. Is H a normal subgroup or not? How do you see this from the covering space?

(2) Prove that if M is a compact 3-dimensional submanifold of S^3, then $H_1(M; \mathbb{Z})$ is torsion-free.

(3) Prove that a continuous mapping from the 17-dimensional unit ball to itself fixes some point.

(4) (a) Describe a cell decomposition of $\mathbb{R}P^n$ involving one cell of each dimension from 0 to n inclusive.
   (b) Write down the associated cell chain complex of $\mathbb{R}P^5$ with $\mathbb{Z}$